Date

Expt. No. The Galilean Transformations Page No. 1

The Consequences of research work of Galileo on the Motion of the Projectile led him to formulate Galilean Transformations.

These are used to relate the motions which are observed by two observers in two different inertial frames.

Some of his results are as follows:

1.) The Motion of a Particle Projected at any angle may be derived from the Motion of the Particle Thrown vertically upward.

2.) If a Particle is Thrown straight up from a cart which is Moving with uniform speed, The observer on the cart may see the Particle Moving up and down, but the Motion observed by an observer on the ground may be described by superimposing the Motion of the cart into that of Projectile.

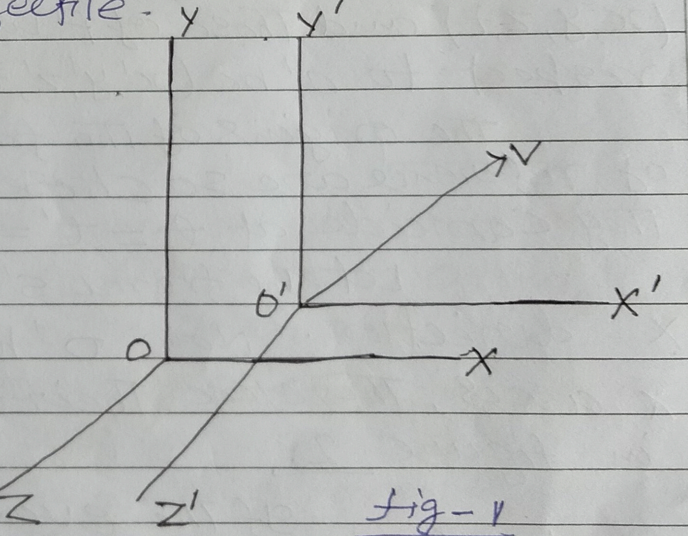

fig-1

Consider two frames of reference one at rest and other Moving with constant speed with respect to that at rest.

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Suppose that there are two observers observing the event at any point P from the two frames of reference simultaneously.

Let the two frames of reference be parallel to each other that is x' axis is parallel to x axis, y' axis parallel to y axes and z' axis parallel to z axis.

Case 1

When the second frame is moving relative to first along +ve X-axis.

Consider two frames S and S' of references one at rest and the other is moving with uniform velocity V.

Let O and O' be the observers situated at the origins of S and S' respectively. They are observing the same event at any point P.

Let the coordinates of P with respect to O as origin be (x, y, z, t) and those of P with respect to O' be (x', y', z', t').

The origins of the frames of reference are so chosen that they coincide at $t = t' = 0$

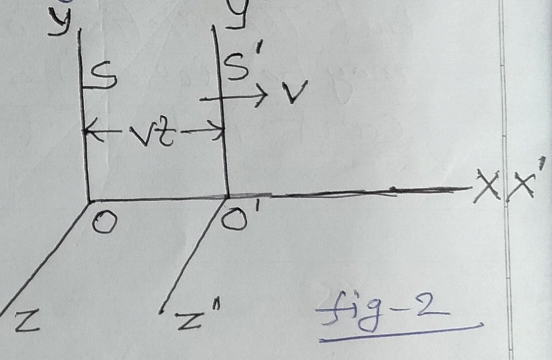

fig-2

Let the frame S' have the velocity V only in x'-direction. Now O' has velocity V only along X axis, therefore their x axes will coincide as in figure 2.

If the event is happening at P at any particular time and the observations are taken by both the observers, then the two systems can be related by the equation.

$$\left.\begin{aligned} x' &= x - vt \\ y' &= y \\ z' &= z \\ t' &= t \end{aligned}\right\} \quad \text{——— (1)}$$

These are the Galilean transformation equations relating the observations of position and time made by two observers in two different inertial frames.

Case –II

When the second frame is moving along a straight line relative to first along any direction:

Consider two frames S & S', the latter moving with velocity v relative to the former S, such that

$$\vec{V} = i\vec{V_x} + j\vec{V_y} + k\vec{V_z}$$

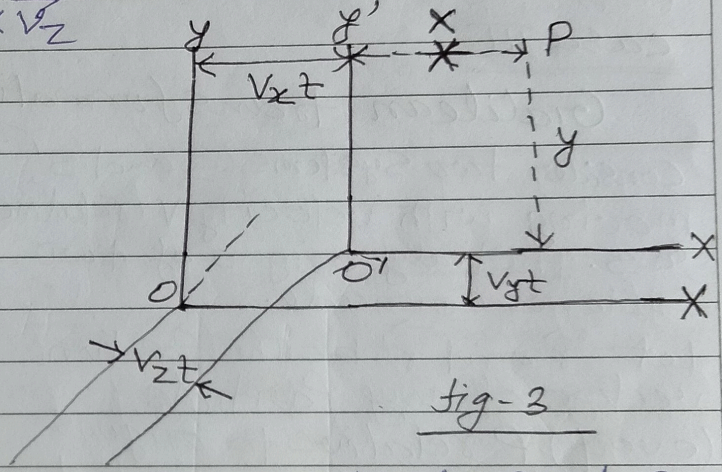

fig-3

where v_x, v_y and v_z are the components of v along X, Y, and Z axis respectively.

Let O and O' be the observers situated at the origins of S & S' respectively observing the same event at point P.

Let the coordinates of P relate to O and O' be (x, y, z, t) and (x', y', z', t') respectively.

The origins and axes of the two frames are so chosen that they coincide at $t = t' = 0$.

From fig-3 the Z-axis is perpendicular to the plane of the paper. Then after a time t, the frame S' is separated from frame S by a distance $v_x t$, $v_y t$, and $v_z t$ along x, y and z axes respectively as shown in figure-3.

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Then the observations of the event at P taken by both the observers may be seen to be related, referring to fig. by the following equations.

$$\left.\begin{aligned} x' &= x - v_x t \quad &\text{—(i)} \\ y' &= y - v_y t \quad &\text{—(ii)} \\ z' &= z - v_z t \quad &\text{—(iii)} \\ t' &= t \quad &\text{—(iv)} \end{aligned}\right\} \text{—(2)}$$

These are the Galilean transformations relating to the observations of position and time made by two observers in two different inertial frames.

case III

Galilean transformations in vector form.

Consider two systems S and S' moving with velocity V relative to S. Initially origins of two systems coincide.

Let $\vec{r}$ and $\vec{r}'$ be the position vectors of any particle (event) P relative to origins O and O' of system S and S' respectively after time t. Then $\vec{OO'} = vt$

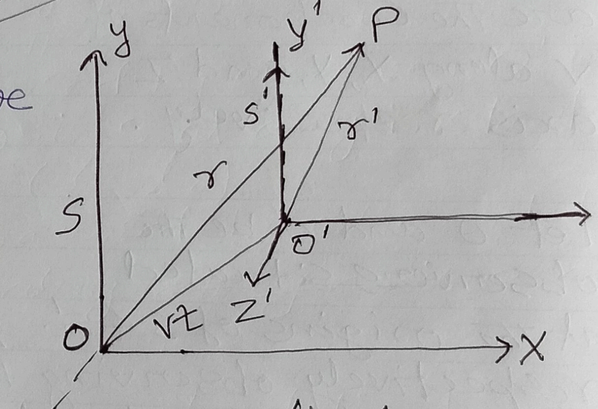

fig. 4

So from fig. 4 by using the law of triangle of vector addition

$$\vec{r} = \vec{r}' + \vec{Vt}$$

or $$\vec{r}' = \vec{r} - \vec{Vt} \qquad \text{—— (3)}$$

Also $$t' = t \qquad \text{—— (4)}$$

These equ^ns represent Galilean transformations of space and time in vector form.

equ^n (1) (2) & (3) are called time-dependent Galilean transformations, since they are time dependent and were obtained by Galileo.